

MULTISHAKER MODAL TESTING

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INTRODUCTION

This report summarizes the research conducted on this contract and reported on in several technical reports and papers. Reference 1 is a technical report summarizing research conducted during Phase I of the project (August 1983 - August 1985), when research was conducted on the following topics:

1. Development of a generalized time-domain component mode synthesis technique for damped systems;
2. Development of a frequency-domain component mode synthesis method for damped systems, and
3. Development of a system identification algorithm applicable to general damped systems.

The goals of Phase II of the project (August 1985 - August 1987) were:

1. To continue development of the multi-input, multi-output system identification method initially studied during Phase I;
2. To explore the possibility of using multishaker testing to identify parameters associated with modeling structures with Lanczos modes;
3. To continue development of the modal test facility in the Structural Dynamics Laboratory by acquiring feedback control hardware; and
4. To design and fabricate a model suitable for studying the control of flexible structures, and to develop associated instrumentation.

This report presents abstracts of the major publications which have been issued on the Phase II topics outlined above.

MULTI-INPUT, MULTI-OUTPUT SYSTEM IDENTIFICATION

During Phase I, research was conducted on multi-input, multi-output modal testing (or system identification). This work was reported on in Refs. 2 and 3. Development of this algorithm was continued during Phase II with the results being published in Refs. 4-6. These references provide complete documentation of the computer program for State-Space Formulation of Multi-Shaker Modal Analysis (SFMSMA), including a user's manual (Ref. 4), fully-documented FORTRAN code (Ref. 5), and a paper presented at the 5th International Modal Analysis Conference (Ref. 6). The principal new features of SFMSMA are: introduction of two automatic procedures for reducing model size (order), generalization of the form of model damping, and utilization of stable least-squares solution techniques.

Basic Frequency-Domain Algorithm

To develop a modal parameter estimation method for general linear, time-invariant dynamic systems, it is assumed that the dynamics of the system can be represented by a discrete analytical model

$$[M]\ddot{\vec{x}}(t) + [C]\dot{\vec{x}}(t) + [K]\vec{x}(t) = [D]\vec{p}(t) \quad (1)$$

where $[M]$, $[C]$, and $[K]$ are, respectively, the mass, damping, and stiffness matrices of order $N \times N$; $[D]$ is an $N \times N_p$ force distribution matrix; and $\vec{p}(t)$ represents the N_p shaker (exciter) forces. $\vec{x}(t)$ is the vector of N displacements, which is assumed to include displacements at all exciter coordinates and all response measurement coordinates. The order, N , required to completely describe the dynamics of a complex structure is theoretically infinite, but the response of the structure at a limited number of measurement coordinates due to excitation over a limited frequency bandwidth can be represented by a reduced-order model. The vector $\vec{x}(t)$ may be "condensed" to a reduced-order vector $\vec{\gamma}(t)$, and $\vec{x}(t)$ can be "reconstructed" from $\vec{\gamma}(t)$ by the transformations

$$\vec{\gamma}(t) = [\psi_C]\vec{x}(t) \quad (2)$$

$$\vec{x}(t) = [\psi_R]\vec{\gamma}(t) + \vec{\epsilon}(t) \cong [\psi_R]\vec{\gamma}(t) \quad (3)$$

A reduced-order frequency-domain model is created by applying the Fourier transform to the time histories in Eq. (1) and then employing Eqs. (2) and (3). The final result is

$$\ddot{\vec{\gamma}}(\omega) + [\hat{C}]\left(\frac{1}{j\omega}\right)\dot{\vec{\gamma}}(\omega) + [\hat{K}]\left(\frac{1}{-\omega^2}\right)\ddot{\vec{\gamma}}(\omega) = [\hat{D}]\vec{p}(\omega) \quad (4)$$

Let $[\ddot{\vec{x}}(\omega_i)]$ represent a set of N_ω measured acceleration response vectors at frequencies $\omega_1, \omega_2, \dots, \omega_{N_\omega}$. Then, reduced acceleration response spectra may be obtained by using Eq. (2) to get

$$[\ddot{\vec{\gamma}}(\omega_i)] \equiv [\ddot{\vec{\gamma}}(\omega_1)\ddot{\vec{\gamma}}(\omega_2)\dots\ddot{\vec{\gamma}}(\omega_{N_\omega})] = [\psi_C][\ddot{\vec{x}}(\omega_i)] \quad (5)$$

Finally, Eq. (4) may be converted to the following form, from which reduced system matrices $[\hat{C}]$, $[\hat{K}]$, and $[\hat{D}]$ can be estimated using a least-squares procedure.

$$[\hat{C}][\hat{K}][\hat{D}] \begin{bmatrix} \left[\frac{1}{j\omega_i} \ddot{\vec{\gamma}}(\omega_i) \right] \\ \left[\frac{1}{-\omega_i^2} \ddot{\vec{\gamma}}(\omega_i) \right] \\ \left[-\vec{p}(\omega_i) \right] \end{bmatrix} = [-\ddot{\vec{\gamma}}(\omega_i)] \quad (6)$$

The identified matrices $[\hat{C}]$ and $[\hat{K}]$ are then employed in an eigensolution which determines the eigenvalues (damped natural frequencies and decay rates) and complex eigenvectors (complex normal modes).

Coordinate Reduction and Least-Squares Solution Strategies

The accuracy achieved in modal parameters for a particular reduced-order model hinges upon the selection of $[\psi_C]$ and $[\psi_R]$. Two specific reduction strategies are implemented in the SFMSMA computer program: and “independent coordinate method” based on Gaussian elimination, and a “principal component method.”

A widely used reduction technique in finite element analysis and structural dynamics is referred to as Guyan reduction (Ref. 7). It assumes that a subset of coordinates may be expressed as a linear combination of the remaining coordinates, i.e. if $\vec{x}(t)$ is partitioned into independent and dependent coordinates

$$\vec{x}(t) = \begin{bmatrix} \vec{x}_i(t) \\ \vec{x}_d(t) \end{bmatrix} \quad (7)$$

then the dependent coordinates are given by

$$\vec{x}_d(t) = [\psi_{di}]\vec{x}_i(t) \quad (8)$$

Then, the transformations $[\psi_C]$ and $[\psi_R]$ in Eqs. (2) and (3) become

$$[\psi_C] = [[I][0]], \quad [\psi_R] = \begin{bmatrix} [I] \\ [\psi_{di}] \end{bmatrix} \quad (9)$$

Equation (8) forms the basis for least-squares estimation of the transformation matrix $[\psi_{di}]$ using acceleration spectra.

$$[\ddot{\vec{x}}_d(\omega_k)] = [\psi_{di}][\ddot{\vec{x}}_i(\omega_k)] \quad (10)$$

Equations (9) and (10) thus define the “independent coordinate” procedure for model order reduction.

The second model order reduction strategy provided in the SFMSMA system identification program is principal component analysis. Principal component analysis seeks transformation matrices $[\psi_C]$ and $[\psi_R]$ such that the expected value of the inner product of the error term in Eq. (3), i.e.

$$E \{ \tilde{\epsilon}^T(t) \tilde{\epsilon}(t) \} \quad (11)$$

is minimized. In addition to producing the transformation matrices $[\psi_C]$ and $[\psi_R]$, the approach also provides a measure for use in estimating the appropriate order for the reduced-order model.

Least-squares solutions of linear algebraic equations are required at several points in the SFMSMA algorithm, e.g. for determining $[\hat{C}]$, $[\hat{K}]$ and $[\hat{D}]$ from Eq. (6) and for determining $[\psi_{di}]$ from Eq. (10). The least-squares solution techniques provided are the singular value decomposition (SVD) method and the method of Householder transformations.

Summary

A modal parameter estimation procedure applicable to linear, time-invariant systems has been developed. The technique permits multiple non-coherent input excitations to be applied to the structure. A consistent set of modal parameters is produced by the algorithm, which is formulated such that no assumptions concerning the form of the linear damping matrix $[C]$ need be made. Even in the presence of relatively high noise levels, accurate damped frequencies, decay rates and complex modes have been identified for a system with both skew-symmetric and non-proportional damping terms.

Since many practical modal analyses involve numerous measurement locations, two different transformation strategies for reducing the size of the model were investigated. Both approaches attempt to minimize the amount of required user interaction. For the small sample size employed in several example runs, the independent coordinate method proved to be more accurate than the principal component method. The latter, however, directly provides an estimate of the order of model required to accurately model system dynamics in a given frequency range. Two least-squares solution procedures which are noted for their accuracy and stability, the singular value decomposition technique and Householder’s method, were evaluated. Both methods were found to be accurate and stable for this application. Due to its faster execution time and relative ease of use, Householder’s method appears to be the preferred method.

SYSTEM IDENTIFICATION BASED ON UNSYMMETRIC BLOCK-LANCZOS MODELING

The SFMSMA system identification algorithm described above first identifies reduced system damping and stiffness matrices $[\hat{C}]$ and $[\hat{K}]$ (which are normalized so that $[\hat{M}] = I$), and these are employed in a standard eigensolution to determine system complex modes, frequencies, and damping factors. Several years ago, research by Wilson (Ref. 8) and Clough (Ref. 9) and their colleagues suggested that use of Lanczos vectors, instead of normal modes, could result in more accurate and more efficient reduced-order dynamic response calculations. This raised the question of whether Lanczos models would also be useful for other structural dynamics applications such as component synthesis and control of flexible structures, and whether the parameters associated with the reduced-order Lanczos models could be obtained experimentally through some modification of system identification programs such as SFMSMA. A companion research project (Contract NAS9-17254 with NASA Lyndon B. Johnson Space Center) has looked at component synthesis and control applications (Refs. 10, 11), while identification of Lanczos models was considered under the present MSFC contract.

Although it has not been possible to complete the development of a system identification program based on Lanczos modeling, important preliminary steps have been completed. As noted above, the SFMSMA method permits the identification of reduced-order *modal* models of systems with general linear damping. To revise this algorithm so that Lanczos models of structures with general linear damping may be identified from experimental measurements, it was first necessary to extend Lanczos methods for modeling undamped structures (e.g. Refs. 8 and 9) to accommodate general linear damping. The development of an unsymmetric block-Lanczos algorithm for application to the dynamics of structures having general linear damping, described in Ref. 12, will now be summarized.

Unsymmetric Block-Lanczos Algorithm

The second-order equations of motion of the system, Eq. (1), may be converted to first order and written in the form

$$[A]\ddot{\vec{Z}}(t) + \dot{\vec{Z}}(t) = \vec{F}(t) \quad (12)$$

Reference 12 shows that $[A]$ can be reduced to block-tridiagonal form $[T]$,

$$[F]^{-1}[A][F] = [T] = \begin{bmatrix} [M_1] & [G_1]^T & & & \\ [B_1] & [M_2] & [G_2]^T & & 0 \\ & \cdot & \cdot & \cdot & \\ 0 & \cdot & \cdot & \cdot & [G_{r-1}]^T \\ & & & [B_{r-1}] & [M_r] \end{bmatrix} \quad (13)$$

where

$$[F] \equiv [X_1, \dots, X_r], \quad [X_i] \in \mathbb{R}^{n \times p} \quad (14)$$

$$[F]^{-T} \equiv E \equiv [Y_1, \dots, Y_r], \quad [Y_i] \in \mathbb{R}^{n \times p} \quad (15)$$

$$[M_i] \in \mathbb{R}^{p \times p}$$

$$[B_i], [G_i] \in \mathbb{R}^{p \times p} \text{ and upper-triangular matrices}$$

$$r : \text{ number of blocks}$$

$$p : \text{ size of blocks}$$

The columns of $[F]$ are referred to as *right Lanczos vectors*, while the columns of $[E]$ are referred to as *left Lanczos vectors*.

The unsymmetric block-Lanczos algorithm for computing $[T]$ may be summarized as follows:

$$[X_1], [Y_1] \in \mathbb{R}^{n \times p} \text{ given, with } [X_1]^T [Y_1] = [I_p]$$

$$[M_1] = [Y_1]^T [A] [X_1]$$

For $j = 1, \dots, r-1 (rp \leq n)$

$$[R_j] = [A][X_j] - [X_j][M_j] - [X_{j-1}][G_{j-1}]^T \in \mathbb{R}^{n \times p} \quad ([X_0][G_0]^T \equiv 0)$$

$$[P_j] = [A]^T [Y_j] - [Y_j][M_j]^T - [Y_{j-1}][B_{j-1}]^T \in \mathbb{R}^{n \times p} \quad ([Y_0][B_0]^T \equiv 0) \quad (16)$$

$$[L_j][U_j] = [P_j]^T [R_j] \quad (L-U \text{ decomposition})$$

$$[B_j] \equiv [U_j], \quad [X_{j+1}] = [R_j][U_j]^{-1}$$

$$[G_j] \equiv [L_j]^T, \quad [Y_{j+1}] = [P_j][L_j]^{-T}$$

$$[M_{j+1}] = [Y_{j+1}]^T [A] [X_{j+1}]$$

Validation Studies

To test the validity of reduced-order models generated using the unsymmetric block-Lanczos algorithm described above, both eigensolutions and forced-response solutions were computed for the example 8DOF Beam-Rotor Assembly shown in Fig. 1.

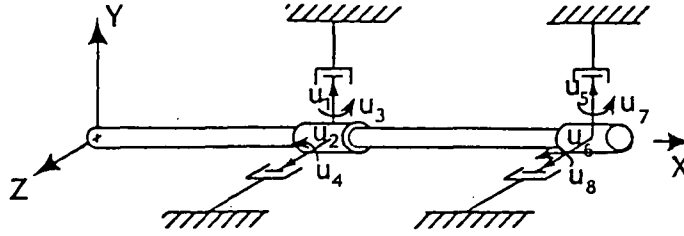


Figure 1: 8-DOF Beam-Rotor Assembly.

Results of these studies are presented in Ref. 12. As a sample of these results, Fig. 2a shows the random external force applied at DOF 5, while Fig. 2b shows the resulting response at DOF 5. While the original system has 8DOF (16 states), a reduced-order Lanczos model with only 4 states accurately describes the response if the starting vectors in $[X_1]$ are static deflection vectors based on unit loads at the excitation coordinates rather than just arbitrary starting vectors.

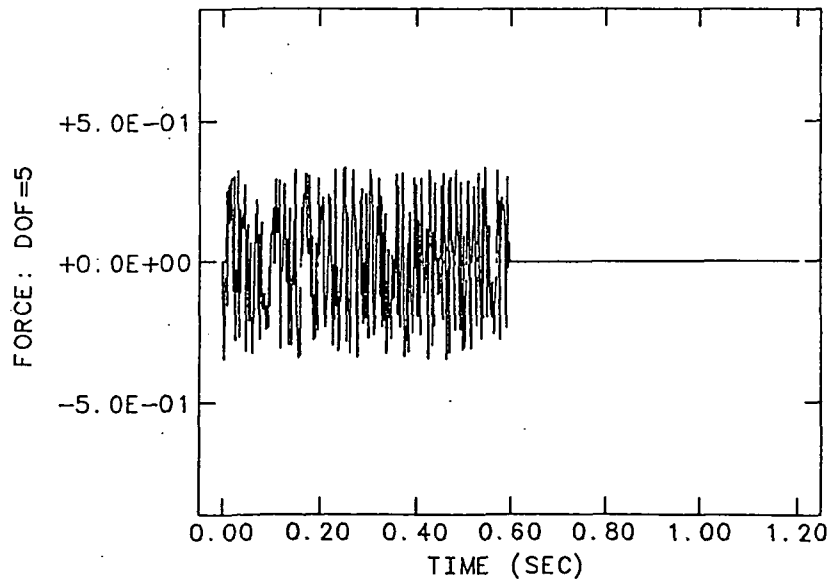


Figure 2a: External Random Force.

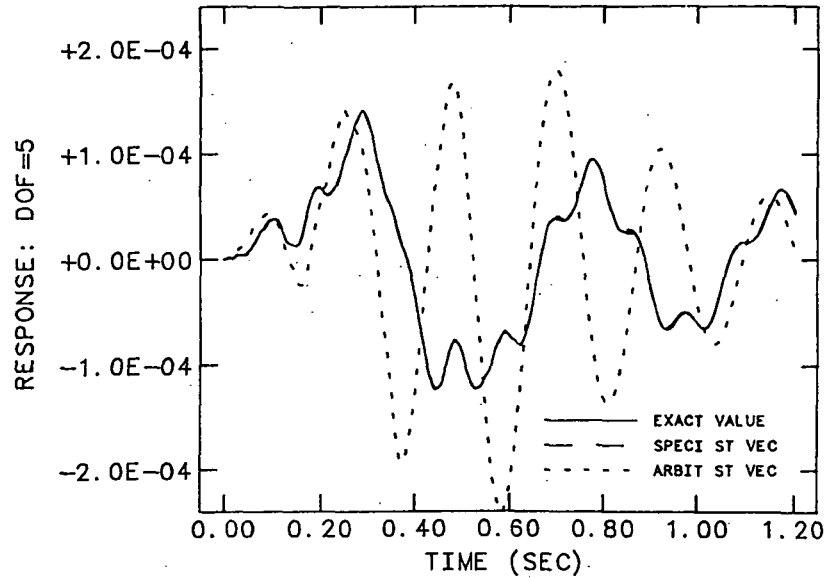


Figure 2.b: Response; Order = 4 (2 x 2).

Summary

An unsymmetric block-Lanczos algorithm has been developed and tested. This algorithm can form the basis for future attempts to use experimental "modal analysis" to identify the parameters of reduced-order Lanczos models. The algorithm works well for systems having repeated eigenvalues and for systems having general linear damping. Computation time can be significantly reduced by use of the Lanczos models generated by this algorithm.

CONTROL-STRUCTURE-INTERACTION (CSI) STUDIES

While the previous two sections of this report have described research consisting primarily of analytical studies aimed at development of algorithms for system identification and reduced-order system modeling, the present section describes the development of a control-structure-interaction facility and use of this facility for conducting a CSI experiment on a flexible beam test structure.

Hardware Acquisition

The principal hardware acquisitions during the course of this project were a Systolic Systems PC-1000 array processor and an Acoustic Power Systems Model 113 ELECTRO-SEIS shaker and Model 114 DUAL-MODE power amplifier. Funding for these was provided by The University of Texas at Austin. NASA funds originally intended for use in upgrading the PC-1000 were transferred to Salaries and Wages in order to continue the support of graduate research assistants beyond the original contract termination date. This did not adversely impact the research project.

CSI Hardware and Instrumentation

Reference 13 describes in detail the development of a control-structure-interaction facility including hardware acquisition, design and fabrication of specialized hardware and instrumentation, and design and fabrication of a flexible beam test structure. Figure 3 is a block diagram of the entire CSI system including actuator (APS-113), sensors (accelerometers, velocity transducers, displacement transducers, strain gauges), system identification hardware GenRad 2515), digital control processor (Systolic Systems PC-1000), and miscellaneous instrumentation and hardware. Figure 4 shows the configuration of the hanging beam test structure. As indicated on this figure, a special cross-flexure was employed to simulate a "pinned-end" condition at the top of the beam. However, a detailed finite element model of the flexure was required in order to model the support condition accurately. Figure 5 shows the APS-113 shaker with displacement and velocity transducers to provide co-located feedback signals, an accelerometer for use in cancellation of the mass of the shaker armature, and a force sensor mounted on a cross-flexure type stinger. Figure 6 shows the GenRad 2515, the Systolic Systems PC-1000, and several instrumentation "black boxes" used for summing signals and controlling the electromagnetic "initial condition" release device.

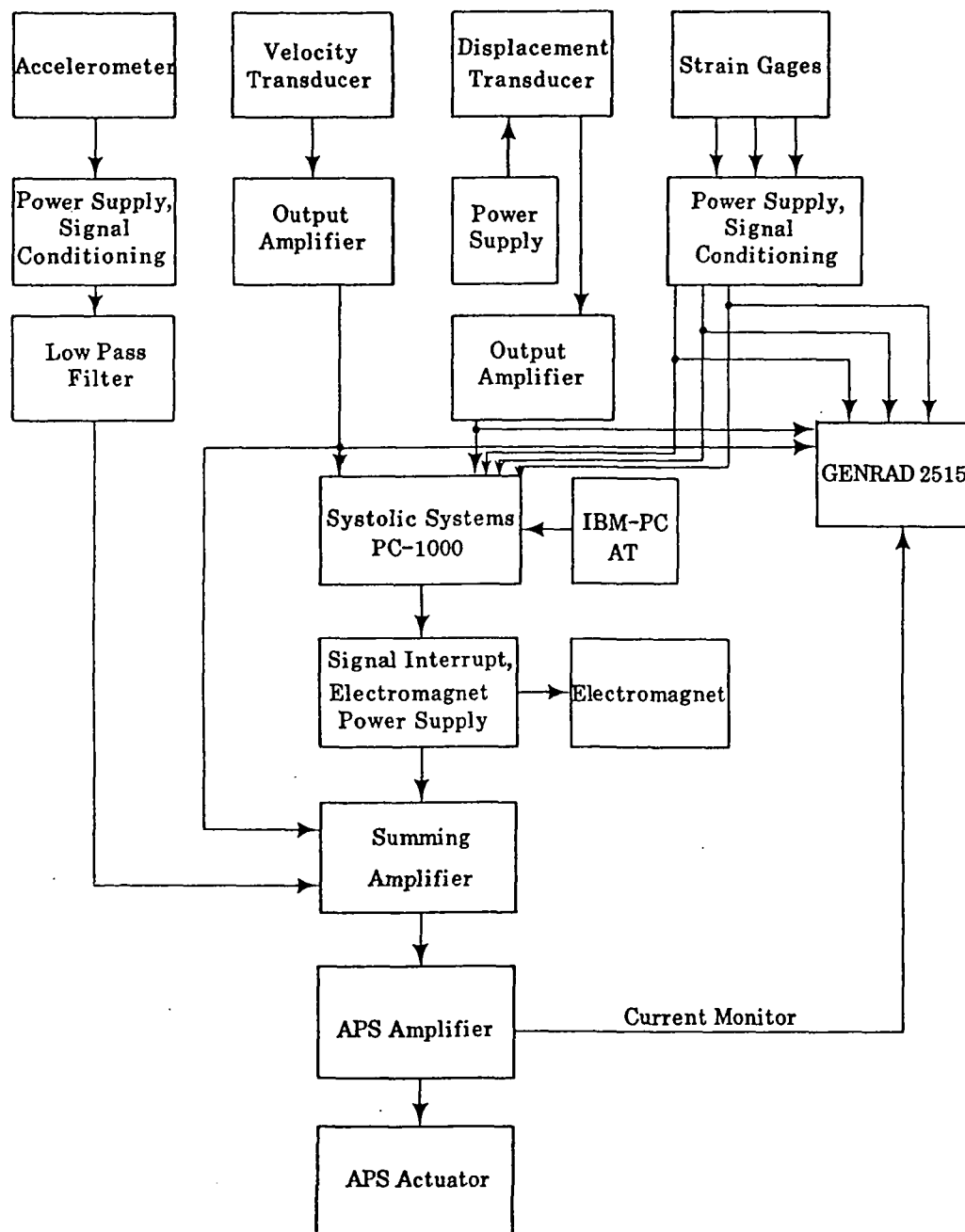


Figure 3: System Block Diagram

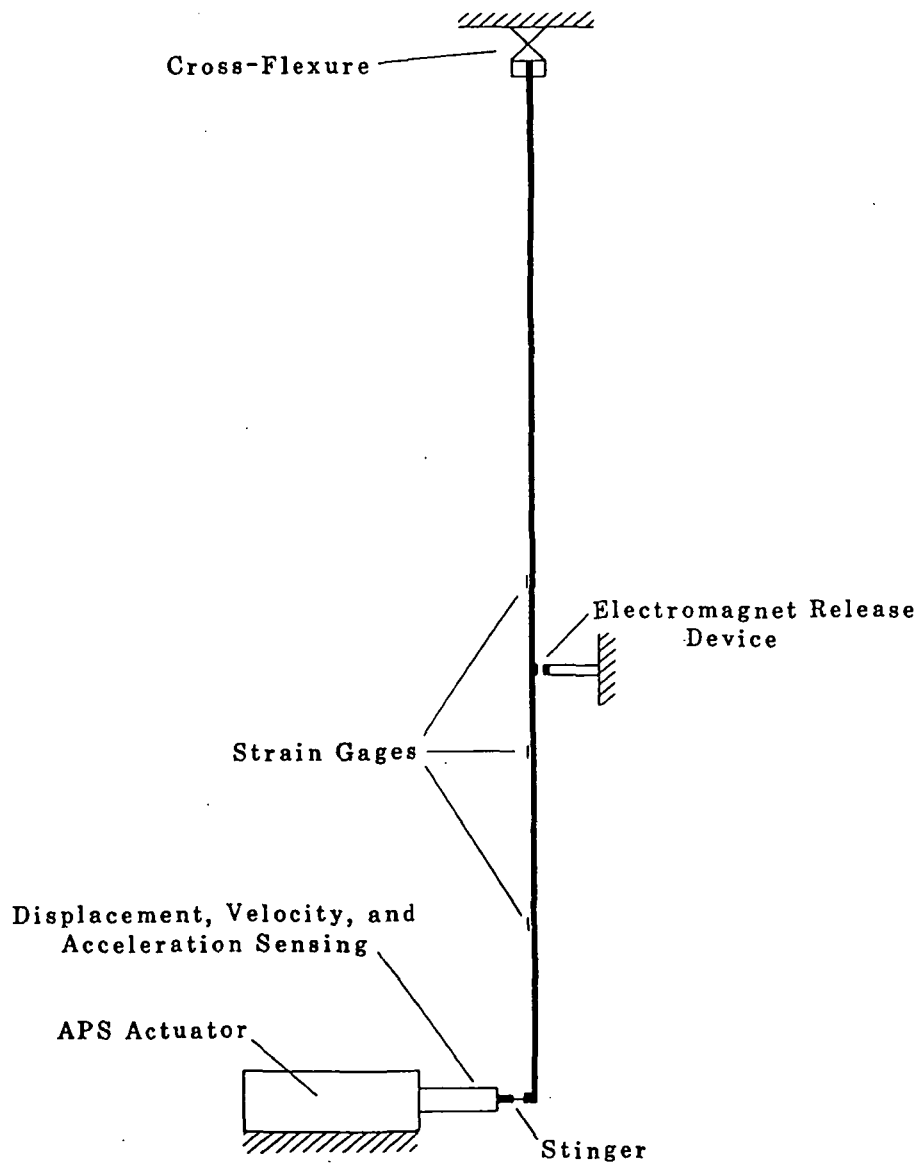


Figure 4: Test Structure

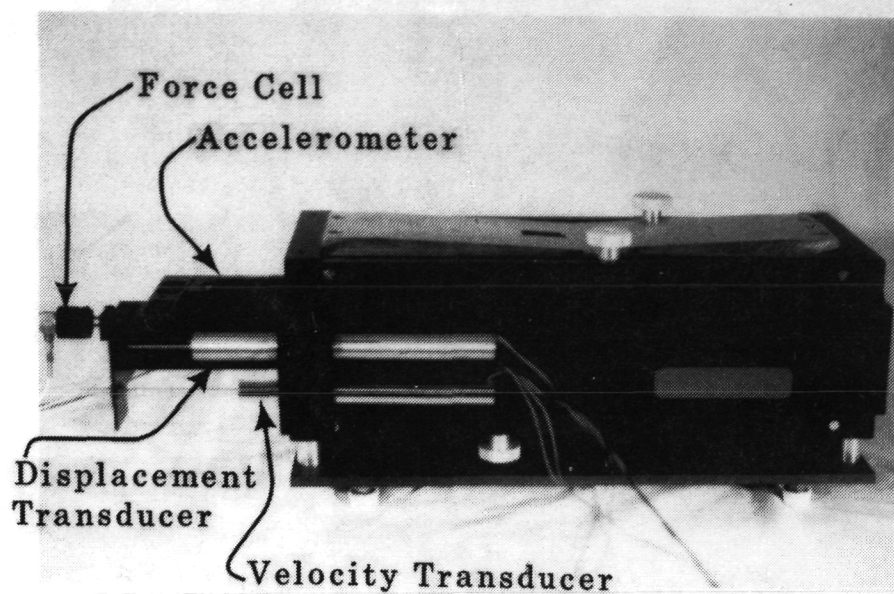


Figure 5: Shaker with Transducers

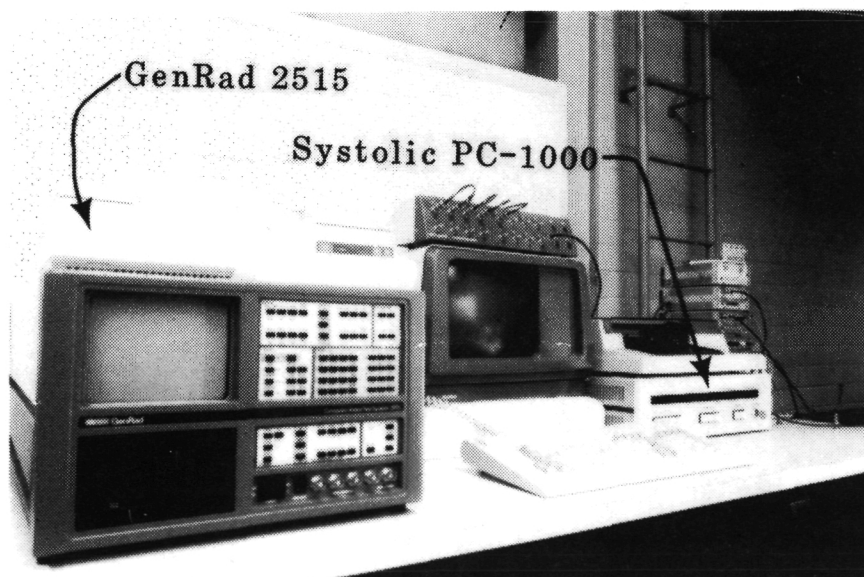


Figure 6: GenRad 2515, Systolic Systems PC-1000, and Special Instrumentation

CSI Experiment

The CSI experiment consisted of the following steps:

1. Fabricate the test structure (hanging beam illustrated in Fig. 4) including support flexures and release mechanism;
2. Attach actuator, sensors, and release mechanism;
3. Create a detailed finite element model of the beam and cross-flexure support;
4. Perform an eigensolution to obtain modes and frequencies of the analytical model;
5. Perform an experimental modal analysis to confirm the analytical model;
6. Develop an optimal feedback controller design based on a single force input and multiple outputs including displacement, velocity and strain;
7. Implement the control using the PC-1000 array processor.

Table 1 shows the first five natural frequencies obtained from the analytical (finite element) model and from the modal test. By including a detailed model of the cross-flexure support rather than just assuming a “pinned-end” condition at the cross-flexure point, it was possible to obtain agreement to within 2% on the first five modes. (Four modes were later used in designing a reduced-order controller.)

Mode	Mathematical (Hz)	Experimental (Hz)	Percent Difference
1	0.311	0.305	1.96
2	1.796	1.813	0.94
3	5.402	5.375	0.50
4	11.030	10.875	1.43
5	18.605	18.375	1.25

Table 1: Natural Frequencies

Linear Quadratic Regulator (LQR) design methods were used to determine the control gains for the Single-Input, Multiple-Output (SIMO) system. The input $u(t)$ is the electrodynamic shaker force, and the measurement (output) vector consists of various combinations of the tip displacement and velocity (co-located with input) and the strain measurements. The optimal control force is given by

$$u(t) = -[\tilde{E}][\tilde{C}]^{-1}\tilde{y} \quad (17)$$

where \tilde{y} is the measurement vector

$$\tilde{y} = \begin{bmatrix} x_T \\ \epsilon_{g1} \\ \epsilon_{g2} \\ \epsilon_{g3} \\ \dot{x}_T \\ \dot{\epsilon}_{g1} \\ \dot{\epsilon}_{g2} \\ \dot{\epsilon}_{g3} \end{bmatrix} \quad (18)$$

$[\tilde{C}]$ is determined from the analytical modal responses at the sensors, and $[\tilde{E}]$ is the optimal modal feedback gain matrix derived from LQR theory and computed using the PC-MATLAB computer program (Ref. 14).

Summary

A control-structure-interaction facility has been developed. The facility includes various actuators and sensors, a digital control array processor, and a hanging beam test structure. All of the steps in a flexible structure control experiment (analytical modeling, verification of analytical model, hardware design and fabrication, calibration, etc.) were conducted except for the final step of closed-loop feedback control. This step was not completed due to a DC bias problem in the PC-1000 controller. Once this problem is corrected the facility should permit many meaningful, CSI experiments to be conducted.

OVERVIEW OF LANCZOS MODELING IN STRUCTURAL DYNAMICS

An overview paper describing several studies of applications of Lanczos methods in structural dynamics has been prepared (Ref. 15). Included in this paper is a brief description of the unsymmetric block-Lanczos studies described above and in Ref. 12. Also described in this paper are methods for component synthesis based on Lanczos (or Krylov) modes and for feedback control based on Lanczos modes.

CONCLUSIONS AND RECOMMENDATIONS

Several significant contributions are contained in the reports and papers prepared during Phase II of Contract NAS8-35338 and abstracted above. The principal contributions are:

1. The refinement and documentation of a multi-input, multi-output modal parameter estimation algorithm which is applicable to general linear, time-invariant dynamic systems.
2. The development and testing of an unsymmetric block-Lanczos algorithm for reduced-order modeling of linear systems with arbitrary damping.
3. The development of a control-structure-interaction (CSI) test facility in the Structural Dynamics Laboratory of The University of Texas at Austin.

Further research on the following topics is suggested:

1. Combine the unsymmetric block-Lanczos modeling algorithm with the modal parameter estimation algorithm to produce a system identification algorithm which identifies a reduced-order Lanczos model.
2. Continue development of the CSI facility and use it to explore methods for controlling structures having many closely-spaced-frequency modes and methods for controlling multi-component bodies.

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